Ch. 8 Risk and Rates of Return

Topics

- Measuring Return
- Measuring Risk
- Risk & Diversification
- CAPM

Return, Risk and Capital Market

- Managers must estimate current and future opportunity rates of return for investment evaluation.
- Estimating the opportunity rate begins with a study of historical rates of return on varying risk investments.
- The level of risk and required rate of return are directly related. Investors require higher rates of return for increased risk.

Investment returns

The rate of return on an investment can be calculated as follows:

$$\text{Return} = \frac{(\text{Amount received} - \text{Amount invested})}{\text{Amount invested}}$$

For example, if $1,000 is invested and $1,100 is returned after one year, the rate of return for this investment is:

($1,100 - $1,000) / $1,000 =

Returns

- **Dollar Return** = cash received + change in value of the asset in dollars
- **Percentage Return** = (cash received + change in value of the asset)/original investment

$$\text{percentage return} = \frac{\text{dollar return}}{\text{beginning market value}}$$

$$= \frac{\text{dividend} + \text{change in market value}}{\text{beginning market value}}$$

$$= \text{dividend yield} + \text{capital gains yield}$$

Returns: Example

- Suppose you bought 100 shares of Timber Inc. one year ago today at $25. Over the last year, you received $20 in dividends (= 20 cents per share × 100 shares). At the end of the year, the stock sells for $30. How did you do?
- Dollar gain:
- Percentage gain for the year:

Measuring Return

- Holding period return: Return that an investor would get when holding an investment over a period of *n* years.

$$r_{HR} = (1 + r_1)\times(1 + r_2)\times (1 + r_3)\times \ldots \times(1 + r_n) - 1$$

- Arithmetic return: Average return

$$r_A = (r_1 + r_2 + r_3 \ldots + r_n)/n$$

- Geometric return

$$r_G = \sqrt[n]{(r_1+1)(r_2+1)(r_3+1)\ldots(r_n+1)} - 1$$
$$= [(r_1+1)(r_2+1)(r_3+1)\ldots(r_n+1)]^{(1/n)} - 1$$

Holding Period Return: Example

- Suppose your investment provides the following returns over a four-year period:

Year	Return
1	10%
2	-5%
3	20%
4	15%

Holding period return

=

Example (continued)

$$\text{Arithmetic average return} = \frac{r_1 + r_2 + r_3 + r_4}{4}$$

$= \underline{\hspace{4cm}} =$

- However, an investor who held this investment would have actually realized an annual return of 9.58%:
  44.21% = (1 + .095844)^4 - 1

Geometric average return:

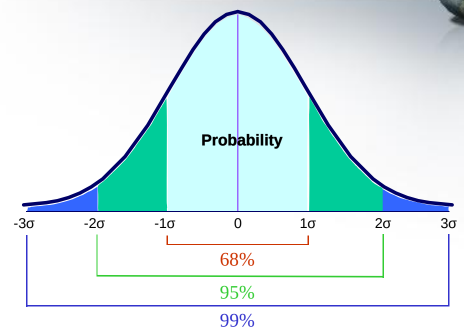
$(1 + r_g)^4 = (1 + r_1) \times (1 + r_2) \times (1 + r_3) \times (1 + r_4)$

$r_g =$

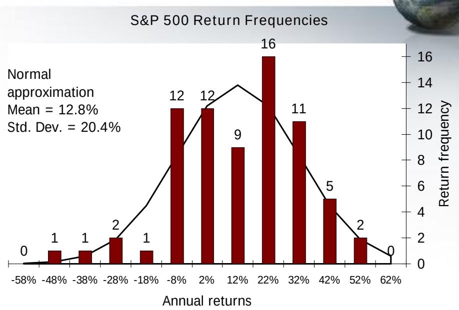
Return Statistics

- The history of capital market returns can be summarized by describing:
  - Average return
  - The standard deviation of those returns
  - The frequency distribution of the returns

Return - Normal Distribution

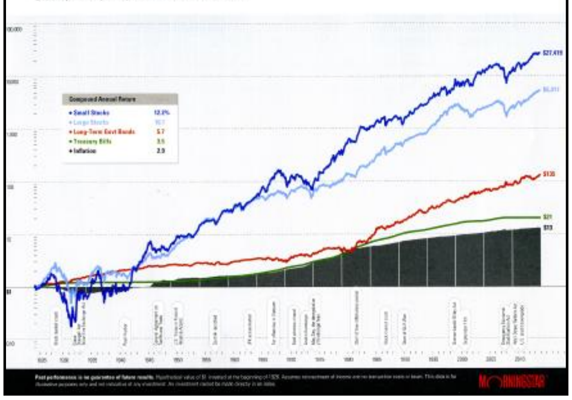


Return - Normal Distribution

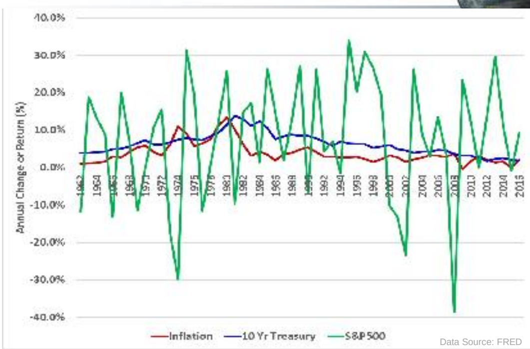


Source: © Stocks, Bonds, Bills, and Inflation 2000 Yearbook™, Ibbotson Associates, Inc., Chicago (annually updates work by Roger G. Ibbotson and Rex A. Sinquefield). All rights reserved.

Ibbotson® SBBI®
Stocks, Bonds, Bills, and Inflation 1926–2014



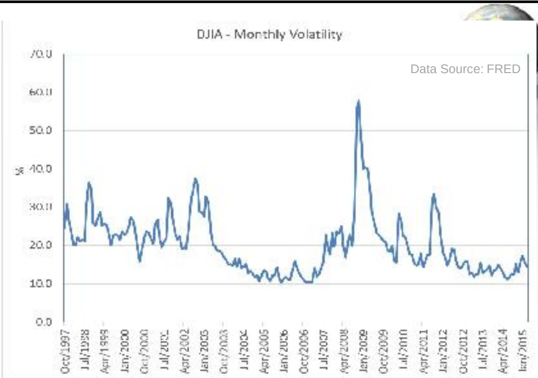
Rates of Return 1963-2016



Measuring Risk

- There is no universally agreed-upon definition of risk.
- **Risk = Volatility?**
- Variance: Average value of squared deviations from mean. A measure of volatility.
- Standard Deviation: Average value of squared deviations from mean. A measure of volatility.

$$SD = \sqrt{VAR} = \sqrt{\frac{(R_1 - \bar{R})^2 + (R_2 - \bar{R})^2 + \cdots (R_T - \bar{R})^2}{T-1}}$$



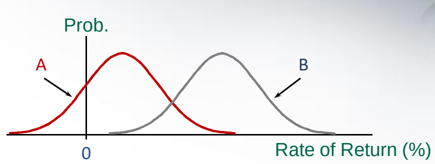
The volatility of stocks is not constant every year.

Coefficient of Variation (CV)

- A standardized measure of dispersion about the expected value, that shows the risk per unit of return.

$$CV = \frac{\text{Standard deviation}}{\text{Expected return}} = \frac{\sigma}{\hat{r}}$$

CV as a Measure of Relative Risk

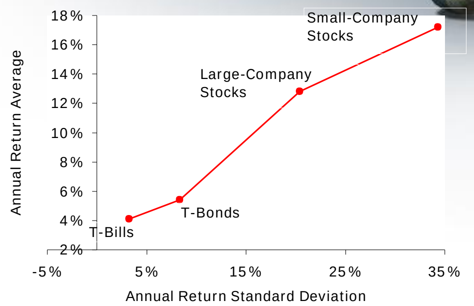


$\sigma_A = \sigma_B$, but A is riskier because of a larger probability of losses. In other words, the same amount of risk (as measured by σ) for smaller returns.

Risk Premium and Risk-Free Returns

- **Risk Premium:** Additional return (over and above the risk-free rate) resulting from bearing risk.
  - Rate of return on T-bills is essentially risk-free.
  - Investing in stocks is risky, but there are compensations.
  - Risk premium on Stocks = Return on stocks - Return on T-bills
- One of the most significant observations of stock market data is this long-run excess of stock return over the risk-free return.

The Risk-Return Tradeoff



Risk and Return

- Stocks have outperformed bonds over most of the twentieth century, although stocks have also exhibited more risk.
- The stocks of small companies have outperformed the stocks of large companies over most of the twentieth century, again with more risk.
- **Higher return ⇔ Higher risk**

Expected Return

$\boxed{E[R] = \bar{R} = \sum_{s=1}^{N} p_s R_s}$ ⇨ Expected Return

Example:

Outcome-s	Probability-P_s	R_A	R_B
Boom	.25	20%	5%
Normal	.50	10%	10%
Bust	.25	0%	15%

Expected return of a security

$E(R_A)$ = (.25)(.20)+(.50)(.10)+(.25)(.00) =

$E(R_B)$ =

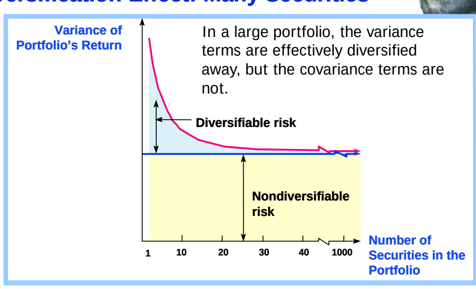
Variance, Std. Deviation & Covariance

- Variance of security A = $E(R_A - E(R_A))^2$
  σ^2_A =
  σ^2_B =
- Standard deviation = $(\text{Variance})^{1/2}$
  σ_A =
- Covariance = $E[(R_A - E(R_A))(R_B - E(R_B))]$
  σ_{AB} =
- Correlation = $\sigma_{AB} / (\sigma_A \sigma_B)$
  ρ =

Diversification vs. Risk

- Diversification: Holding many stocks in one's portfolio. For simplicity, we consider a portfolio of two stocks.
- How do we measure portfolio risk? The risk of such a portfolio can be measured by the variance of the portfolio returns.
- Unique Risk: Risk factors affecting only that firm. Also called "diversifiable risk."
- Market Risk: Economy-wide sources of risk that affect the overall stock market. Also called "systematic risk" or "non-diversifiable risk."

Diversification Effect: Many Securities



Thus, diversification can eliminate some, but not all of the risk of individual securities.

Diversification vs. Risk

- Expected Return of portfolio:
  $E(R_P)$ = (Weight on A) $\times$ $E(R_A)$ + (Weight on B) $\times$ $E(R_B)$
- Variance of Portfolio:
  σ^2_P = (Weight on A)^2 $\times$ σ^2_A + (Weight on B)^2 $\times$ σ^2_B
  + 2 $\times$ (Weight on A) $\times$ (Weight on B) $\times$ ρ $\times$ σ_A $\times$ σ_B

Diversification vs. Risk

Suppose you invest $100 in security A and $200 in security B.
Dollar returns under each outcome are as follows:

Outcome	Prob	Return on $100 in A	Return on $200 in B	Total return	Return on $300 in A&B
Boom	.25	$120	$210	$330	10%
Normal	.50	$110	$220	$330	10%
Bust	.25	$100	$230	$330	10%
Return		10%	10%	10%	
Variance		.00500	.00125	.0	
Std. Dev.		.07071	.03536	.0	

Portfolio variance:
σ^2_P =

Efficient Sets and Diversification

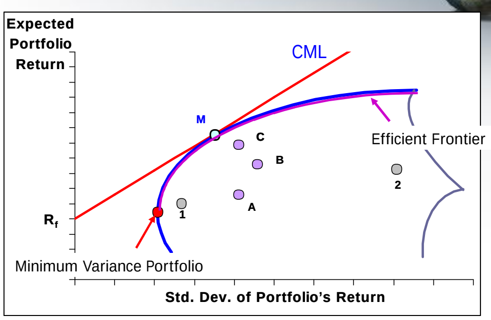
- Correlation & Efficient Sets for Two Assets
- The expected return on a portfolio is the weighted average of the expected returns on the individual securities.
- As long as $\rho < 1$, the standard deviation of a portfolio of two securities is less than the weighted average of the standard deviations of the individual securities.

⇨ Diversification effect occurs whenever the correlation between the two securities is below 1.

Diversification

- Feasible Set vs. Efficient Set
- Minimum Variance Portfolio
- Efficient Frontier
- Capital Market Line

Capital Market Line and Efficient Frontier



Capital Asset Pricing Model - CAPM

- The capital asset pricing model (CAPM) provides us with an explicit expression for the equilibrium expected returns on all assets in terms of risk-free rate, a market return per unit of risk and the riskiness of each asset.
- The CAPM was originally developed by Sharpe [1964], Lintner [1965], Treynor [1961] and Mossin [1966].

Capital Asset Pricing Model - CAPM

The basic form of the CAPM is an expression for the equilibrium expected returns on all assets:

$$E(R_j) = R_f + \beta_j \times (E(R_M) - R_f)$$

where,

- $E(R_j)$: the expected returns on the j^{th} security
- R_f: risk-free rate
- $E(R_M)$: the expected return on a market portfolio
- $(E(R_M) - R_f)$: market risk premium
- $\beta_j = cov(r_j, R_M) / \sigma^2(R_M)$

Capital Asset Pricing Model - CAPM

- The contribution of a security to the risk of a well-diversified portfolio is proportional to the covariance of the security's return with the market's return. This contribution is called the beta.
- The CAPM states that the expected return on a security is positively related to the security's beta.

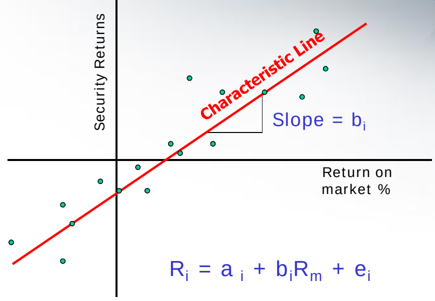
→ Note: Clearly, the estimate of beta will depend upon the choice of a proxy for the market portfolio.

$$\boxed{\begin{aligned} E(R_j) &= R_f + \beta_j \times (E(R_M) - R_f) \\ \beta_j &= \frac{cov(r_j, R_M)}{\sigma_M^2} \end{aligned}}$$

Comments on beta

- If beta = 1.0, the security is just as risky as the market.
- If beta > 1.0, the security is riskier than the market.
- If beta < 1.0, the security is less risky than the market.
- Most stocks have betas in the range of 0 to 3.

Estimating β with regression



$R_i = a_i + b_i R_m + e_i$

Important Properties of the CAPM

- Systematic risk
  - ✓ In equilibrium, every asset must be priced so that its risk-adjusted required rate of return falls exactly on the security market line.
  - ✓ Investors can diversify away all risk except the covariance of an asset with the market portfolio - the systematic risk.
- Portfolio betas
  - ✓ The measure of risk for individual assets is linearly additive when the assets are combined into portfolios. So, portfolio betas, β_P, are linearly weighted combinations of individual asset betas.

  $_p$ = (Weight on A) × $_A$ + (Weight on B) × $_B$

CAPM - Example

- Consider two stocks, Stock A with beta of 1.5 and Stock B with a beta of 0.7. The risk-free rate is 7% and the risk premium is 8.5%.

  $E(R_A)$ = .07 + 1.5 × .085 =

  $E(R_B)$ =
- Construct a portfolio by investing equally in the two stocks

  $E(R_P)$ =

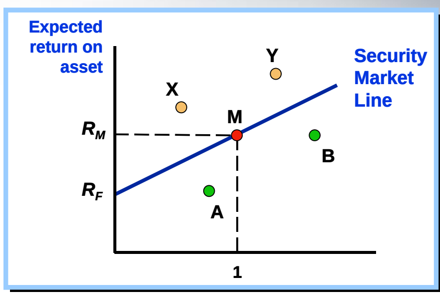
  β_p = (Weight on A) × β_A + (Weight on B) × β_B

  =
- Check:

  $E(R_P)$ = R_f + $\beta_j \times [E(R_m) - R_f]$

  =

CAPM and the Security Market Line



Capital Asset Pricing Model - CAPM

- Basic assumptions of the Sharp-Lintner-Mossin derivation of the CAPM are:

1. All investors are single-period expected utility of wealth maximizers whose utility functions are based on the mean and variance of return.
2. All investors can borrow or lend an indefinite amount at the risk free rate, and there are no restrictions on short sales.
3. All investors have homogeneous expectations of the end-of-period joint distribution of returns.
4. Securities markets are frictionless and perfectly competitive. All investors are price takers.
5. No taxes and regulations.

CAPM - Conclusion

- The CAPM was shown to provide a useful conceptual framework for capital budgeting and the cost of capital.
- Even though the CAPM is often rejected by empirical tests, its main implications are upheld:
  - systematic risk, β, is a valid measure of risk,
  - the model is linear, and
  - the trade-off between return and risk is positive.